# Intro to Discrete Structures

* Logical Equivalence – if two propositions have matching truth table values, they are logically equivalent.
* If either one is true, return true – inclusive or ∨
* Exclusive or, if both propositions are true, return false. If only one is true, return true. - + with circle around it
* Logical connective – → P implies q

Means if p, then q

P is sufficient for q

Q is necessary for p

* P → q is the same thing as –q → ¬p
* The latter is called a contrapositive of p implies q.
* Converse of p→q is q → p
* Bidirectional implication = p <-> q

If and only if

Or “iff”

* P → Q is the same as ¬p ∨ q
* Tautology – always true no matter the input
* Contradiction – always false no matter the input
* Contingency – neither tautology or contradiction
* T0 - denotes a compound statement that is always true
* F0 – denotes a compound statement that is always false
* Rule of Substitution Suppose we have a composite statement, P, containing some sub-statement, q. If q = q ‘ to create a new statement, P ‘ and p = p ‘
* For example p v (q v r) We know (q v r) = (r v q)  
  so p v (q v r) = p v (r v q)
* Some common questions:  
  Simplify a compound proposition.  
  Show that two compound statements are logically equivalent.  
  Show a compound statement is a tautology or contradiction.  
  Negate and Simplify
* ¬((p v q) → r) Given
* ¬(¬(p v q) v r) Definition of implication on (1)
* ¬¬(p v q) ^ ¬r DeMorgan’s Law on (2)
* (p v q) ^ ¬r Double negation
* Notation ∃x p(x) – Existential Quantifier
* There exists a value of x such that p(x)
* ∀x p(x) – For all x, p(x) is true.
* Open statement – there is a variable that hasn’t been defined, not a statement until there is a value.
* D(x) = x is delicious.
* Universe of discourse = All muffins Μ.
* ∀x d(x) = “All muffins are delicious”
* Not true! Bran muffins!
* ¬∀x d(x) – “Not all muffins are delicious”  
   “There exists at least one muffin that is not delicious.”
* ∃x[p(x) ^ q(x)] – there exists a value that satisfies both p and q of x.
* You can use implications with inequalities.
* Nested Quantifiers:
* M^2 + N^2 = 41. Let U = Z
* There is some integer M and N that make this statement true.
* Important laws: Negation of quantifiers
* This law basically means that putting a ¬ in front of ∀ or ∃, it just switches the letter.
* A variable is bound when we apply a quantifier to it. All the variables in an expression must be bound to make it a statement. (aka, a proposition)
* Open statements have two parts:
* P is predicate, x is the variable
* Predicate logic / predicate calculus – logical
* Argument – sequence of statements that end in a conclusion
* Premises are the causes
* Conclusion is the result
* P1 ^ p2 ^ ….. pn → q
* Premises → conclusion
* Rules of Inteference

Modus Ponens (Rule of Detachment)

P

P → q

∴q

Modus Tollens

¬q

p→q

∴¬p

Note: this is modus ponens + equivalence of contrapositive

Law of Syllogism

p → q

q → r

∴p → r

Disjunctive Syllogism

P v q

¬p

∴q

Addition (Disjunctive Amplification)  
p

∴p v q

Disjunction means or, amplification means expanding

Simplification (conjunctive simplification)

p ^ q

∴p

(conjunction means and)

Rule of Conjunction

p

q

∴ p ^ q

Resolution

p v q

¬p v r

∴ q v r

WARNING: When applying a rule of inference to a premise, it must match the whole form of that premise.

Ex:

1. (r v s) → (t v u)
2. (t v u) → (v v w)

We can apply: p → q, q → r, therefore p → r (law of syllogism)

(r v s) → (v v w)

Ex:

(This is bad!)

1. s v (p →q)
2. q → r

Cannot use law of syllogism

No answer. Ugly fruit.

Ex: Verify the following argument

p → q

q → (r ^ s)

¬r v (¬t v u)

p ^ t

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∴ v → u

Proof:

1. p → q Premise
2. q → (r ^ s) Premise
3. p → (r ^ s) Syllogism on (1) and (2)
4. p ^ t Premise
5. p Conjunctive Simplication on (4)
6. (r ^ s) Modus Ponens on (5) and (3)
7. r Conjunctive Simplication on (6)
8. ¬r v (¬t v u) Premise
9. (¬t v u) Disjunctive Syllogism on (7) and (8)
10. t Conjunctive Simplification
11. u Disjunctive syllogism on (10) and (9)
12. ¬v v u Disjunctive Amplification
13. v → u Definition of implication
14. Q.E.D v → u

“Show” (It can’t be shown, it’s not true)

p v q

∴p

1. P v q Premise
2. ¬¬(p v q) Double Negation
3. ¬(¬p ^ ¬q) De Morgans on (2)
4. ¬(¬p) Conjunctive Simplification on (3) - can’t do this because it doesn’t apply to the whole premise. That’s why this is wrong.
5. p Double negation

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∀x (p(x) → q(x))

∀x (q(x) → r(x)) Syllogism

Can’t use syllogism because we don’t have the premises matching the whole form of the rule.

Rules of Inference for Quantifiers

Universal Instantiation

1. ∀x p(x)

∴ p(c)

where c is an arbitrary element from U

Universal Generalization

1. p(c) is true for arbitrary c from U

∴ ∀x p(x)

Existential instantiation

1. ∃x p(x)

∴ p(c) for some particular c (not arbitrary)

Existential generalization

1. p(c) for some particular c (not arbitrary)

∃x p(x)

Example: All CS professors have studied discrete math.

John is a CS professor. Therefore John has studied discrete math.

Let p(x): x is a CS professor

Q(x): x has studied discrete math.

U: All people

Show: ∀x (p(x) → q(x))

P(John)

∴ q(John)

Proof

1. ∀x (p(x) → q(x)) Premise
2. p(John) → q(John) Universal Specification on (1)
3. p(John) Premise
4. q(John) Modus Ponens on (2) and (3)

Q.E.D

* Disjoint Sets are like unique sets.
* Symmetric difference is the set A and the set B but not A and B (exclusive OR)
* A set is a collection in a universe of discourse.
* Cartesian Product – A x B =

A = {1, 2, 3}

B = {x, y}

A x B = {(1, x), (1, y)…(2, x), (2, y), (3, x), (3, y)

Set Membership Tables

1 = x ε A

0 = ¬ xε A

|  |  |  |
| --- | --- | --- |
| A | B | C |
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |

A c B = ∀ x(x ε A → x ε B) ^ ∃x(x ε B ^ ¬xεA)

A = B = (A ⊆ B ^ B ⊆ A) = ∀x( x ε A ↔ x ε B)

A ∪ B = {x | x ε A v x ε B}

A ⊆ B = ∀x (x ε A → x ε B)

A ⊆ B and B ⊆ C, then A ⊆ C

∀x (x ε A → x ε C)

Let x ε A. Since A ⊆ B, x ε B.

Thus, since B ⊆ C, x ε C.

Product Rule – get all combinations of ordered pairs from two sets.

(elements of first set) \* (elements of second set) = all combinations.

Rule of Sum – Add all possibilities from different elements.

Subtraction Rule – to pick one element from the union, you must subtract the intersection of the two sets.

| A ∪ B | = |A| + |B| - |A∩B|

Division Rule-

If there is an equals sign between two sets, they are both subsets of each other.